Particle Kalman Filtering For Ocean State Estimation

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LONG-TERM GOALS

The long-term scientific objective is to develop a fully nonlinear Bayesian approach that generalizes the optimality of ensemble Kalman filter methods to nonlinear systems and can be suitable for large dimensional data assimilation problems. The approach will be tested with realistic applications to ocean data assimilation problems.

OBJECTIVES

The goal of this research is to explore a new direction that can lead to fully nonlinear filters that can perform better than the ensemble Kalman filter (EnKF) methods with highly nonlinear systems at reasonable computational requirements. We aim at proposing, implementing and testing new nonlinear Kalman filters with ocean data assimilation problems in mind. Simple nonlinear dynamical models will be first considered to evaluate the behavior of these new filters and assess their efficiency compared to the EnKF methods.

APPROACH

The basic idea is to represent the probability density function of the system state given the observations by mixture of Gaussian distributions with low-rank covariance matrices to derive fully nonlinear low-rank filters suitable for realistic ocean data assimilation problems.

The solution of the nonlinear data assimilation problem can be determined from the nonlinear Bayesian filtering theory. The filter provides the conditional probability distribution function (pdf) of the system state given all available measurements. Knowledge of the state pdf allows determining different estimates of the system state, as the minimum variance estimate or the

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maximum a posteriori estimate. The optimal nonlinear filter recursively operates as a succession of a correction (or analysis) step at measurement times to correct the state (predictive) pdf using the Bayes' rule, and a prediction step to propagate the state (analysis) pdf to the time of the next available observation. Despite its simple algorithm, the numerical implementation of the optimal nonlinear filter can be computationally prohibitive, even for systems with very few dimensions.

The particle filter (PF) is a discrete approximation of the nonlinear Bayesian filter and is based on point-mass representation (mixture of Dirac distributions), called particles, of the state pdf. In this filter, the particles are integrated forward with the numerical model to propagate the predictive state pdf in time, and their assigned weights are updated every time new observations are available. In practice, the PF suffers from a major problem known as the degeneracy phenomenon where most weights become concentrated on very few particles and hence only a tiny fraction of the ensemble contributes to the average, causing very often the divergence of the filter. The use of more particles helps alleviating this problem over short time periods only, and the most efficient way to get around it is resampling. Besides being computationally demanding, resampling might introduce Monte Carlo fluctuations which can degrade the filter's performance. Additionally, even with resampling, a very large number of particles is still required to accurately describe the continuous pdf of the system state in order to ensure a good behavior of the filter. This makes brute-force implementation of the PF problematic with computationally demanding ocean models.

The Kalman filter (KF) provides the minimum variance solution of the data assimilation problem only when the system is linear and the statistics of the system errors are Gaussian. The Ensemble Kalman filter (EnKF) combines 'good properties' of the PF and the linear Kalman filter. It has the same prediction step as the PF, but it retains the ``linearity aspect" of the Kalman filter in the analysis. More precisely, it applies the Kalman correction step to each particle. It therefore only updates the first two moments of the particles ensemble, and is thus semi-optimal for non-Gaussian (nonlinear) systems. Despite being ``semi-optimal", many recent studies found that the EnKF is more robust than the PF when small-size ensembles are considered because the Kalman-type update of the particles reduces the risk of ensemble degeneracy by pulling the particles toward the true state of the system.

In this work we used mixture of Gaussian distributions as discrete representation of the pdf of the system state in the nonlinear Bayesian filter. A local linearization about each particle then led to a Kalman-type correction step for each particle complementing the usual particle-type correction. The resulting filter basically runs a weighted ensemble of KFs. As in the EnKF, the Kalman-type correction step attenuates the degeneracy of the ensemble, allowing the filter to efficiently operate with small-size ensembles. The PKF is computationally prohibitive for realistic oceanic data assimilation problems. Approaches to alleviate the computational burden of the PKF were proposed and are being tested.

WORK COMPLETED

An approach to use the optimal nonlinear filtering theory was developed for data assimilation into realistic ocean models was introduced. Different low-rank Gaussian filters were proposed implemented, and tested with the strongly nonlinear Lorenz-96 model. It was found that this approach sets a theoretical framework for the stochastic and deterministic ensemble Kalman filtering methods. More precisely, the ensemble Kalman filter and the square-root ensemble Kalman filters can be derived as simplified variants from this approach. It was further found that the ensemble Kalman filter integrates a simplified form of the state pdf while the square-root filters are Gaussian-based filters. Numerical applications were performed to study the filters behavior and evaluate their performances. The new filters' results were evaluated with those obtained with the Ensemble Kalman filter. It was found that nonlinear filters work more efficiently with strongly nonlinear models providing more accurate estimates of the system states.

New tests with the Lorenz-86 model were performed to compare the performance of the ensemble Kalman Filter (EnKF) and the nonlinear particle filters. The first model admits a chaotic vortical mode coupled to a comparatively fast gravity wave mode. To assess the efficiency of the nonlinear analysis step in enhancing the dynamical balance of the filters solution, identical twin assimilation experiments were designed such that the true state is balanced, but the observational errors project onto all degrees of freedom, including the fast modes. EnKF and nonlinear filter capture the variables in slow manifold well since, once the variables are attracted towards the slow manifold, they stay there. Nonlinear filter captures slaved modes better, implying nonlinear jumps in dependent variables are captured better. This suggests that the solution of the nonlinear filters respects the dynamical balance of the system more. Currently we are working on implementing the nonlinear filters with a barotropic vorticity model.

RESULTS

New nonlinear filtering algorithms were developed and are currently being tested. Numerical results suggest that nonlinear filters behave better than the ensemble Kalman filter methods with strongly nonlinear systems, as we observed with the vortex barotropic model. They also respect the dynamical balance of the system state more resulting in more stable predictions. Gaussian mixtures-based nonlinear filter can operate with small number of particles.

IMPACT/APPLICATIONS

This study led to new sequential data assimilation schemes that generalizes the optimality of the ensemble Kalman filter to nonlinear systems. The new filter can be used to assimilate data to highly nonlinear ocean problems.

TRANSITIONS

Theory and algorithms can be made available to Navy scientists.

RELATED PROJECTS